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Comments on a covariant entropy conjecture

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Abstract: Recently Bousso conjectured the entropy crossing a certain light-like hypersurface is bounded by the surface area. We point out a number of difficulties with this conjecture.

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1. Covariant entropy conjecture

Recently Bousso [1] made the interesting conjecture that the entropy S passing through a certain hypersurface bounded by a two-dimensional spatial surface B with area A must satisfy the bound

$$S \leq \frac{A}{4} \,. \tag{1.1}$$

The hypersurface L in question is to be generated by one of the four null congruences orthogonal to B, with non-positive expansion in the direction away from B. The matter fields in the theory are required to satisfy the dominant energy condition. This *covariant entropy conjecture* is motivated by the proposed holographic principle of 't Hooft and Susskind [2, 3], and recent work on attempts to generalize this principle to cosmological backgrounds [4]–[9].

In this note, we point out a number of difficulties with this proposal. We begin by noting the choice of units is such that the fundamental constants satisfy $\hbar = c = G = k = 1$. With appropriate factors of G and $\hbar$ restored the bound becomes $S \leq A/(4\hbar G)$, and we see the bound becomes trivial in the classical limit, for fixed gravitational interactions. If the bound is to hold at all, it must at least hold in the semiclassical limit of quantum gravity coupled to matter.

However, as is well-known, the dominant energy condition (and also the weak energy condition) fails even for free quantum fields. One consequence of the dominant energy condition is that the local energy density appears positive definite. If one computes the expectation value of the normal-ordered energy momentum tensor $\langle\psi|T_{\mu\nu}|\psi\rangle$ in free scalar field theory, for a state ψ that is an admixture of the ground state and a two-particle state, interference terms in the expectation value can lead to negative values for the energy density. As it stands therefore, the conjecture is inconsistent. If one insists on taking the classical limit, the bound becomes trivial. As soon as one goes to the quantum theory, the conditions for the bound to hold are violated, except possibly for a theory with no matter content.

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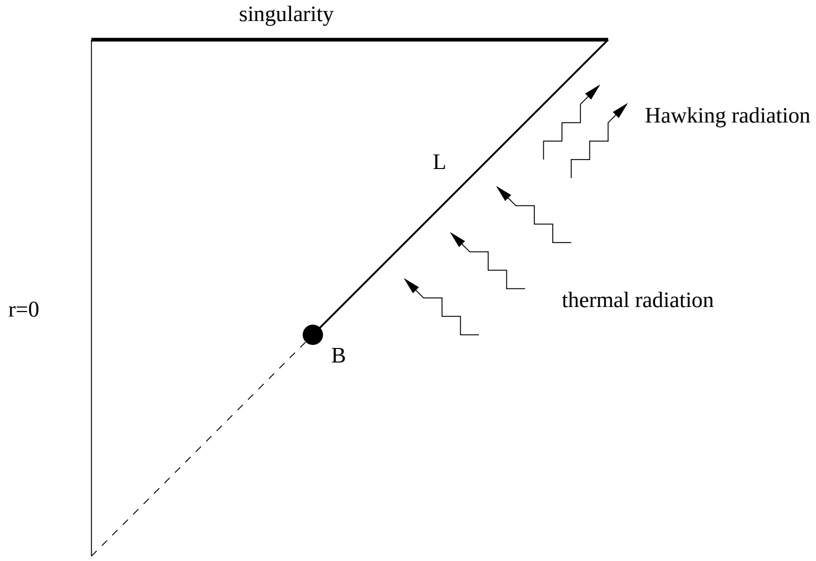

Figure 1: Penrose diagram for a black hole. B is a surface on the event horizon. L is a light-like hypersurface with zero expansion, bounded by B.

One reason for imposing the dominant energy condition was to rule out the possibility of superluminal entropy flow, which would allow for easy violations of the bound, as we see in a moment. In addition, one wants to rule out creating large amounts of entropy with little energy, by simultaneously creating matter with positive and negative energy density. Of course one could try to replace the dominant energy condition with a weaker constraint. However the following example illustrates difficulties with the bound that are independent of this condition.

Consider a black hole in equilibrium with thermal radiation at the same temperature as the Hawking temperature. As discussed in [1], one can take B to be the event horizon at some time, and construct the hypersurface L using future-directed outgoing null generators, as shown in figure 1. Since the black hole's evaporation is supported by the ingoing thermal radiation, the geometry near the horizon is static. One therefore has an infinite amount of time to send a constant flux of entropy across the hypersurface L, in violation of the bound. To be precise, we define the entropy crossing L to be the proper entropy flux, integrated over L. This problem is independent of the matter content of the theory (and hence any energy conditions one might choose to impose), since even for pure gravity, a quantum black hole will Hawking radiate gravitons. Note also that for a large black hole the geometry is such that caustics need not force the hypersurface L to approach the singularity.

It is interesting to consider how this example is consistent with the generalized second law of black hole thermodynamics. While it is true the black hole absorbs an infinite amount of entropy as time goes to infinity, it emits an equal (or larger) amount of entropy in the form of Hawking radiation, in accord with the second law. However the entropy emitted cannot cross the hypersurface L in a causal way. The

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wavelength of Hawking particles is of order the size of the black hole. They are best thought of as originating from outside the black hole, of order the Schwarzschild radius from the event horizon. The Hawking particles themselves do not contribute to the proper entropy passing through L, since they are undetectable to a freely falling observer as she crosses the horizon.

If an arbitrarily large amount of entropy can cross the hypersurface L, how can one regard the log of the number of internal states of the black hole as $A/4$? We will analyze this question in the scenario for the resolution of the black hole information problem discussed in [10]. Roughly speaking, in this picture information crosses the horizon in a completely causal manner, but is effectively transferred to the Hawking radiation in a non-local way as it hits the singularity. Thus we do indeed have superluminal propagation of entropy in this picture. This component of the entropy does not contribute to the local entropy flux passing through L. Although an arbitrarily large amount of entropy does cross L, a low-energy observer inside the black hole could never detect more entropy than $A/4$. In order for an observer inside to live long to detect more entropy than $A/4$ she would have to undergo a trans-planckian acceleration of order e^{M^2}. One sees this via similar gedanken experiments to ones considered in [10, 11]. Likewise an observer entering the horizon at late times will see that most of the entropy has already hit the singularity, preserving the bound on the number of observable internal states.

Having presented perhaps the clearest counterexample to the bound (1.1), we now consider a number of other objections to the arguments Bousso presents as evidence for the bound. Consider a collapsing dust cloud, and construct the hypersurface L as indicated in figure 2, so that it intersects the whole dust cloud. Bousso's claim is that caustics will force L to take a more circuitous route to the singularity, in such a way that the surface L does not intersect the whole dust cloud, preserving the bound (1.1). This relies on the fact that a highly entropic system can never be spherically symmetric so many caustics will be present. Here we simply point out that in the semiclassical approximation, one requires only that the expectation value of the energy momentum tensor be spherically symmetric to avoid caustics. This does not provide a significant constraint on the entropy of such configurations. Beyond the semiclassical limit, the null convergence condition does not hold, so such caustics need not form in the first place. Furthermore, since the construction of the surface L is formulated in terms of classical geometric quantities, it is not clear how Bousso's construction carries over to the full quantum theory.

Much of the other evidence Bousso presents for the bound involves showing consistency with Bekenstein's conjectured bound [12]

$$S \leq 2\pi E R\,, \tag{1.2}$$

in a number of different examples. Here E is the total energy of the system, and R is the circumferential radius, defined as $R = \sqrt{A/4\pi}$ with A the area of the smallest

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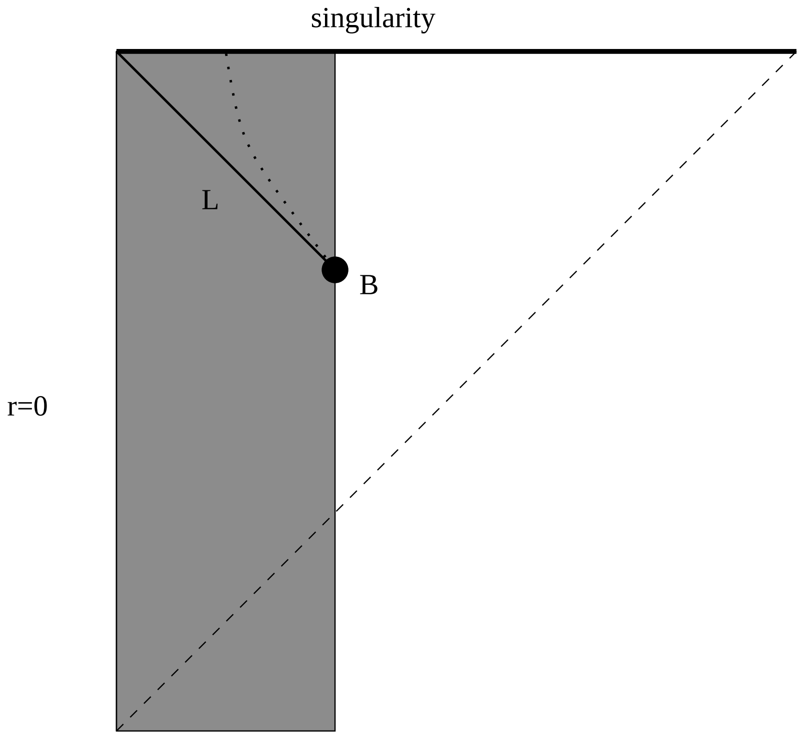

Figure 2: Penrose diagram for collapsing spherical dust cloud. L is light-like hypersurface that intersects all the dust cloud. In order to avoid caustics, L will be deformed to lie along the dotted line.

sphere surrounding the system. This bound has been much discussed since its original proposal [12]. The bound appears to hold for large systems, provided the number of matter species is small [13, 14]. It can easily be violated for sufficiently small systems (for example the free scalar field case already mentioned), for a large number of matter species [13], or for systems at sufficiently low temperature. Likewise, one can take a large number of copies of a small system, to make a large system that violates (1.2). In general, the generalized second law [15] does not imply the bound (1.2) [16, 17]. Some special systems for which (1.2) is violated can be used to construct counterexamples to the covariant entropy bound.

For instance consider a normal region (i.e. not trapped or anti-trapped) in a Friedmann-Robertson-Walker cosmology, as discussed in [1]. The covariant entropy bound implies that the entropy on a spatial hypersurface inside the apparent horizon with radius r_{AH}, should satisfy $S \leq \pi r_{AH}^2$. This follows from the bound (1.2) if this region is treated as a Bekenstein system [1]. Thus the bound (1.1) can potentially be violated for systems which violate (1.2). The simplest example of such a system is a gas of N species of free particles in a box of size R with energy E. The entropy of this gas will diverge as $\log N$. One cannot use this fact to constrain the value of N, as it simply means the bound (1.1) is not a universal bound for any system of matter coupled to gravity.

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2. Conclusions

We have noted a number of difficulties with the current formulation of the covariant entropy conjecture. We propose that in general backgrounds, the only unexpected entropy bounds arise from demanding validity of the generalized second law, as suggested in [5]. This law has passed a number of highly nontrivial consistency checks [16]. However whether the second law gives rise to holographic style bounds is system dependent. It does not constrain the entropy density in the early universe, nor in the final phase of a recollapsing universe. However if an isolated system can collapse to a black hole, the second law implies the entropy satisfies $S \leq A/4$.

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